Mobile Wireless Communication

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2 Characteristics of Radio Environment

2.1 Propagation Models

1. Consider radio waves propagating by two-slope model over the distance under 200 m in Orlando. The average receive power is given by

$$\bar{P}_R = g(d) P_T G_T G_R$$

Assume the attenna gains are both 1 and apply the inverse variation of power with distance for two-slope model we get

$$\begin{aligned}\bar{P}_R &= d^{-n_1}\left(1+\frac{d}{d_b}\right)^{-n_2} P_T\\ \iff P_T &= d^{n_1}\left(1+\frac{d}{d_b}\right)^{n_2} \bar{P}_R\end{aligned}$$

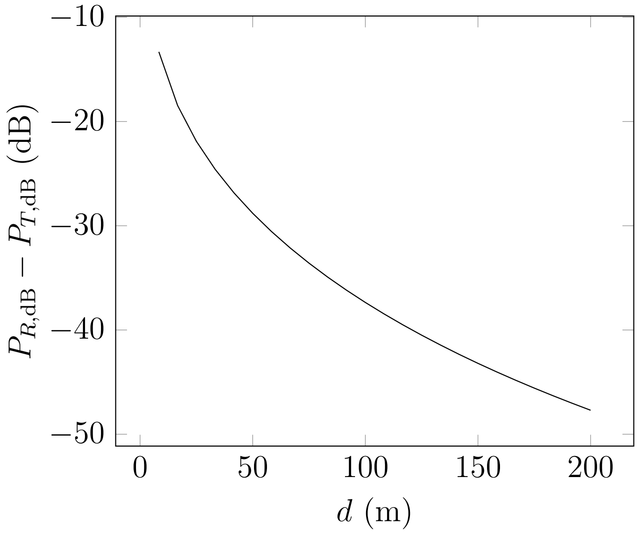
Substituting $d_b = 90\,\mathrm{m}$, $n_1 = 1.3$ and $n_2 = 3.5$ gives us

$$P_T = d^{1.3}\left(1+\frac{d}{90}\right)^{3.5} \bar{P}_R$$

With average power effect experienced, $P_{R,\mathrm{dB}} = 10\lg P_R = 10\lg\bar{P}_R$ and $P_{T,\mathrm{dB}} = 10\lg P_T$ and thus

$$\begin{aligned}P_{T,\mathrm{dB}} - P_{R,\mathrm{dB}} &= 13\lg d + 35\lg\frac{d+90}{90}\\ \iff P_{R,\mathrm{dB}} - P_{T,\mathrm{dB}} &= 35\lg\frac{90}{d+90} - 13\lg d\end{aligned}$$

This is plotted in the figure below



2. Consider a log-normal shadow fading propagation, the receive power is given by

$$P_R = \sqrt[10]{10^X} g(d) P_T G_T G_R$$

where X is a zero-mean normal random variable with STD $\sigma = 6\,\text{dB}$.

(a) Given $\bar{P}_R = 1\,\text{mW}$ at $d = 100\,\text{m}$.

$$P_R > \bar{P}_R \iff \sqrt[10]{10^X} > 1 \iff X > 0$$

Since X is zero-mean and normally distributed, the probability the received power at a mobile at that distance from the base station will exceed $1\,\text{mW}$ is $50\,\%$, and so is the probability it is less than $1\,\text{mW}$.

(b) Let $Y = X/\sigma$, $Y \sim \mathcal{N}(0, 1)$ and $F_X(x) = \Phi(X/\sigma) = \Phi(X/6)$.

The probability a mobile has an acceptable received signal at $10\,\text{mW}$ or higher is

$$\begin{aligned} P\left(P_R \geq 10\bar{P}_R\right) &= P\left(\sqrt[10]{10^X} \geq 10\right) = P(X \geq 10) \\ &= 1 - F_X(10) = 1 - \Phi\left(\frac{10}{6}\right) = 4.78\,\% \end{aligned}$$

(c) For $\sigma = 10\,\text{dB}$, $F_X(x) = \Phi(X/10)$. The probability a mobile has an acceptable received signal at $10\,\text{mW}$ or higher is

$$P\left(P_R \geq 10\bar{P}_R\right) = 1 - F_X(10) = 1 - \Phi(1) = 15.87\,\%$$

(d) If the lower limit for an acceptable received signal is 6 mW, with $\sigma = 6$, the probability a received signal is acceptable is

$$\begin{aligned} P\left(P_R \geq 6\bar{P}_R\right) &= P\left(\sqrt[10]{10^X} \geq 6\right) = P\left(X \geq \lg 6^{10}\right) \\ &= 1 - F_X\left(\lg 6^{10}\right) = 1 - \Phi\left(\frac{\lg 6^{10}}{6}\right) = 9.73\,\% \end{aligned}$$

With $\sigma = 10$, the probability a received signal is acceptable is

$$P\left(P_R \geq 6\bar{P}_R\right) = 1 - F_X\left(\lg 6^{10}\right) = 1 - \Phi\left(\frac{\lg 6^{10}}{10}\right) = 21.82\,\%$$

2.2 Random Channel Characterization

Given $x(t) = e^t * (\Pi(t-1) - \Pi(t-3))$ and $h(t) = \delta(t-1)$, where Π is the rectangular function:

$$\Pi(t) = \begin{cases} 0, & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2}, & \text{if } |t| = \frac{1}{2} \\ 1, & \text{if } |t| < \frac{1}{2} \end{cases}$$

The convolution sum of x and h is

$$\begin{aligned} y(t) &= x(t-1) \\ &= \int_{-\infty}^{\infty} e^{t-z-1}(\Pi(z-2) - \Pi(z-4))\,\mathrm{d}z \\ &= \int_{-\infty}^{\infty} e^{t-z-1}\Pi(z-2)\,\mathrm{d}z - \int_{-\infty}^{\infty} e^{t-z-1}\Pi(z-4)\,\mathrm{d}z \\ &= \int_{1.5}^{2.5} e^{t-z-1}\,\mathrm{d}z - \int_{3.5}^{4.5} e^{t-z-1}\,\mathrm{d}z \\ &= e^{t-1}\left(e^{2.5} - e^{1.5} - e^{4.5} + e^{3.5}\right) \end{aligned}$$

2.3 Fading

1. Consider several delay spreeds D of 0.5 µs, 1 µs and 6 µs.

- For IS-95 and cdma2000 which uses the transmission bandwidth of 1.25 MHz, their symbol interval is 0.8 µs. For the multipath rays to be resolvable, the delay spread must be greater than this (1 µs and 6 µs).
- For WCDMA which uses the bandwidth of 5 MHz, the symbol interval is 0.2 µs, thus symbols are resolvable in all cases.

2. Indicate the condition for flat fading for each of the following data rates with transmission in binary form: 8 kbps, 40 kbps, 100 kbps, 6 Mbps.

Assume information is transmitted in rectangular waves, the symbol interval are 125 µs, 25 µs, 10 µs and 1/6 µs respectively. For flat fading to occur, the delay spread must be significantly less than the symbol interval. Since no data is provided or found, no conclusion is drawn on which radio environments would result in flat fading for each of these data rates.

3 Cellular Concept

3.1 Channel Allocation

1. Assume the simplest path-loss model of $g(d) = d^{-3}$, calculate down-link SIR at point P at the corner of a hexagonal cell in a 3-reuse case.

Using to path-loss model, the signal-to-interference ratio can be approximated from the six first-tier interferers as follows

$$\text{SIR} \approx \frac{1}{\left(\frac{R}{D-R}\right)^3 + \left(\frac{R}{D+R}\right)^3 + 4\left(\frac{R}{D}\right)^3}$$

In a 3-reuse case, $D = \sqrt{3C}R = 3R$, and thus

$$\text{SIR} \approx \frac{1}{\left(\frac{R}{2R}\right)^3 + \left(\frac{R}{4R}\right)^3 + 4\left(\frac{R}{3R}\right)^3} = \frac{1728}{499}$$

2. Calculate the worst-case uplink SIR assuming the co-channel interference is caused only by the closest interfering mobiles in radio cells a distance $D = 3.46R$ away from the cell. Assume the simplest path-loss model of $g(d) = d^{-4}$, the signal-to-interference ratio is approximated by

$$\text{SIR} \approx \frac{P_t/R^4}{6P_t/\left(\frac{3D}{4}\right)^4} = \frac{(3D/4)^4}{6R^4}$$

With $D = 3.46R$ (4-reuse), this becomes

$$\text{SIR} \approx \frac{(3 \cdot 3.46/4)^4}{6} = 7.56$$

3.2 Erlang-B Formula and Sizing a Cell

1. An user who makes a call attempt every 15 minutes, with each call lasts an average of 2 minutes, generate the load of 2/15 erlangs.

2. Consider a mobile system supporting 832 frequency channels and 7-reuse, there are over 118 channels per cell. With the probility of call blocking of $P_B \leq 1\,\%$, the traffic is around 101 erlangs. Given the average call-holding time $h = 200\,\text{s}$, the arrival rate can be calculated to be $\lambda = 0.505\,\text{calls/s}$. Since an user makes a call every 900 s on average, there are approximately 454.5 users. As the density of mobile terminals is $2\,\text{terminals/km}^2$, the area is $227.25\,\text{km}^2$, which indicates a cell radius of $R = 9.35\,\text{km}$, assuming a hexagonal topology.

4 Modulation Techniques

1. Consider communication system operating at the transmission bandwidth of 1 MHz with the rolloff factor of 0.25.

- Achievable data traffic rate is
$$R_s = \frac{B}{1+\beta} = \frac{10^6}{1+0.25} = 800\,\text{kBd/s}$$
- Delay spread that no ISI occurs is much less than the symbol interval, which is $T = B^{-1} = 1\,\mu\text{s}$.
- Using OFDM with $N = 16$ equally spead carriers, for each subcarrier, $\Delta f = 62.5\,\text{kHz}$, $R_s = 50\,\text{kBd/s}$ and $T = 16\,\mu\text{s}$.
- Additionally use 16-QAM, the bit rate is $R_\text{b} = 800\,\text{kb/s}$.

2. Given $B = 1\,\text{MHz}$, $\beta = 0.25$, $R_\text{b} = 4.8\,\text{Mb/s}$ and $T = 25\,\mu\text{s}$.
$R_s = B/(1+\beta) = 0.8\,\text{MBd/s}$, thus 64-QAM is used.
For OFDM, $N = R_s/\Delta f = R_\text{b}T \approx 128$.

3. Consider a transmission of bandwidth $B = 2\,\text{MHz}$, where phase-shift keying and Nyquist rolloff shaping is used.

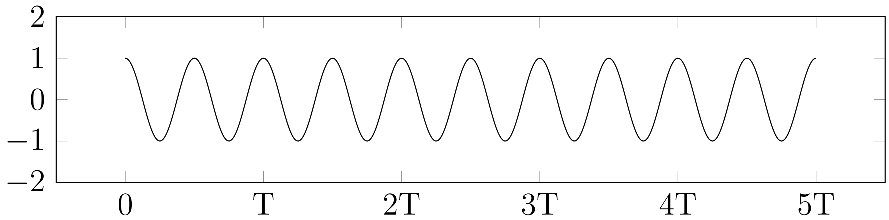
For rolloff factors of 0.2, 0.25, 0.5, the traffic rates are respectively 1.67 MBd/s, 1.6 MBd/s and 1.33 MBd/s.

In order to transmit at a rate of $R_\text{b} = 6.4\,\text{Mb/s}$ when $\beta = 0.25$, 16-QAM should be used.

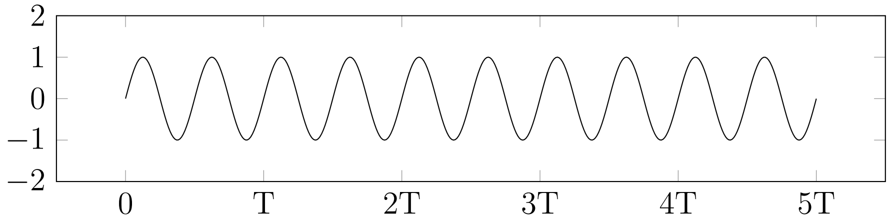
4. Given the input sequence 1001111010 and the following QPSK signal pairs

Successive Signal	a_i	b_i
0 0	-1	-1
0 1	-1	$+1$
1 0	$+1$	-1
1 1	$+1$	$+1$

Let the carrier frequency be some multiple of 1/T

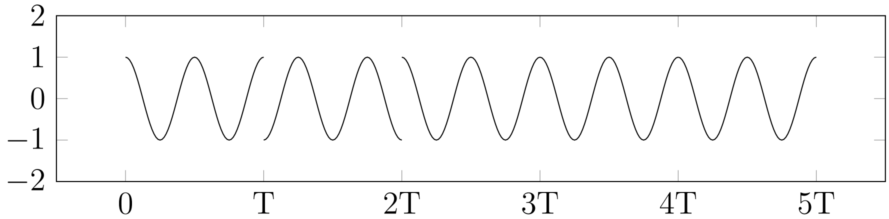


In-phase Carrier

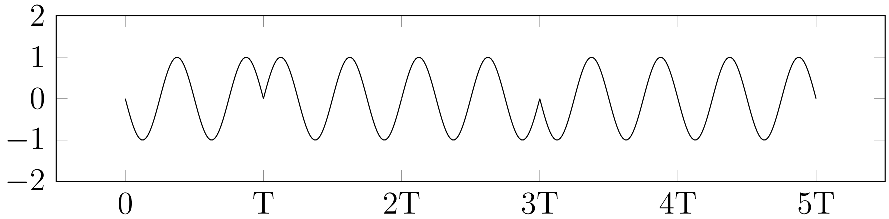


Quadrature-phase Carrier

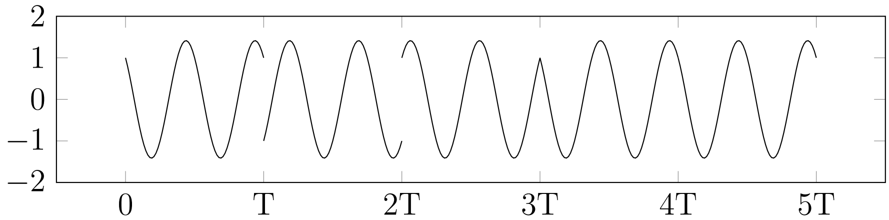
The output QPSK signal would then be



In-phase Component



Quadrature-phase Component



Output Signal

5 Multiple Access Techniques

5.1 Time-Division Multiple Access

Transmission bit rate is the rate at which the bits are transmitted, while the user information bit rate is the rate at which per data are transmitted.

In particular, GSM gives each time slot 576.92 µs, minus 30.46 µs guard time. During this duration, 148 b are tramsmitted, thus the transmission bit rate is $148\,\mathrm{b}/(576.92\,\mu\mathrm{s} - 30.46\,\mu\mathrm{s}) = 270.834\,\mathrm{kb/s}$. Of the 148 b, 114 b are data bits. Furthermore, only one slot per GSM eight-slot frame and 24 out of 26 frames are used to carry information. Therefore, the user bit rate is $114\,\mathrm{b}/4.615\,\mathrm{ms} \cdot 24/26 = 22.8\,\mathrm{kb/s}$.

Similarly, IS-136 has the transmission bit rate of $1944\,\mathrm{b}/40\,\mathrm{ms} = 48.6\,\mathrm{kb/s}$ and $520\,\mathrm{b}/40\,\mathrm{ms} = 13\,\mathrm{kb/s}$.

5.2 Code-Division Multiple Access

Consider IS-95 with the bit rate of 9.6 kb/s and the chip rate of 1.2288 Mc/s, the speading gain is 128 chips per bit.

6 Channel Coding Techniques

6.1 Block Coding

Consider the generator matrix

$$\mathbf{G} = [\mathbf{I}_k \mathbf{P}] = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

it is trivial that $n = 7$, $k = 4$ and

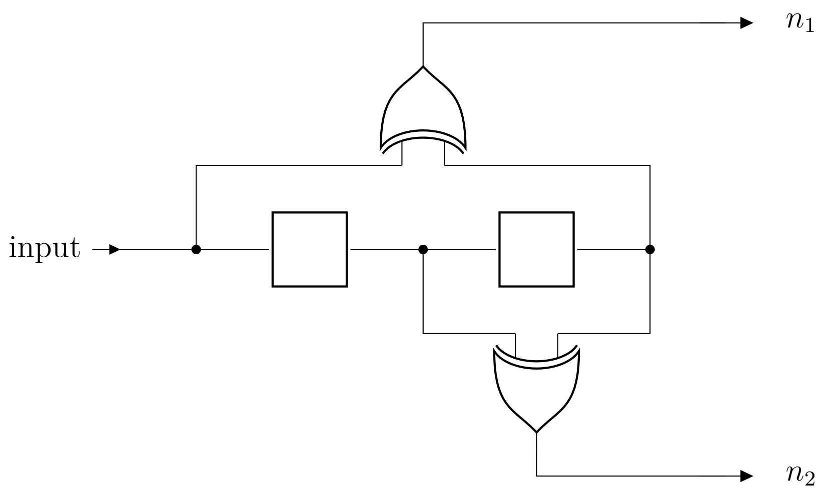
$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

The parity check matrix is then given by

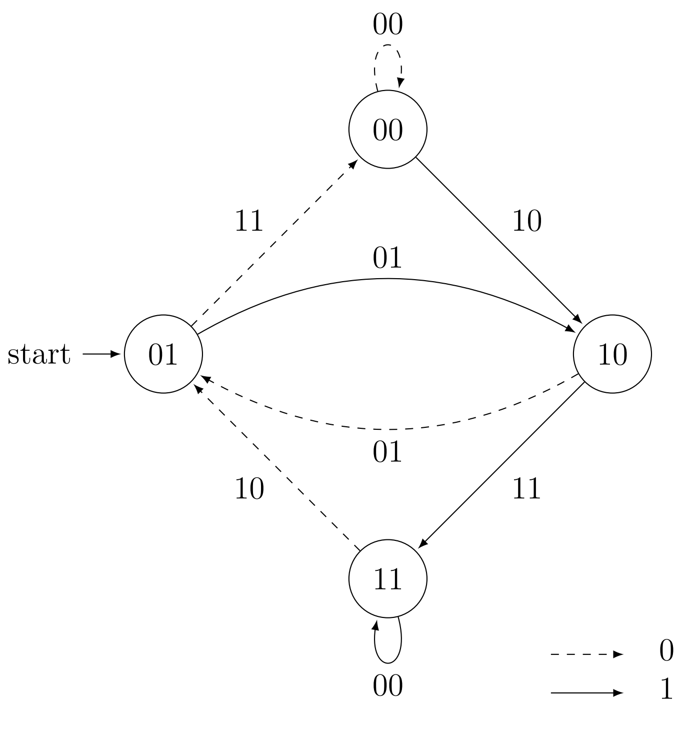
$$\mathbf{H} = [\mathbf{P}^T \mathbf{I}_{n-k}] = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

6.2 Convolutional Coding

Consider a $K = 3$, rate ½ convolution encoder with generators $g_1 = [101]$ and $g_2 = [011]$.



Initialize the encoder with 01, we get the following state diagram



Given the input bit sequence of 10011011, the output would be 0101111011100111.