Numerical Methods: Linear Programming

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November 8, 2019

Given the production contraints and profits of two grades of heating gas in the table below.

	Product		
Resource	Regular	Premium	Availability
Raw gas ($\mathrm{m^3\,t^{-1}}$)	7	11	77
Production time ($\mathrm{h\,t^{-1}}$)	10	8	80
Storage (t)	9	6	
Profit ($\mathrm{t^{-1}}$)	150	175	

(a) Let two nonnegative numbers x_1 and x_2 respectively be the quantities in tonnes of regular and premium gas to be produced. The constraints can then be expressed as

$$\begin{cases}7x_1 + 11x^2 & \le 77\\ 10x_1 + 8x_2 & \le 80\\ x_1 & \le 9\\ x_2 & \le 6\end{cases} \iff \begin{bmatrix}7 & 11\\ 10 & 8\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \le \begin{bmatrix}77\\ 80\\ 9\\ 6\end{bmatrix}$$

The total profit is the linear function to be maximized:

$$\Pi(x_1, x_2) = 150x_1 + 175x_2 = \begin{bmatrix}150\\ 175\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Let $\mathbf{x} = \begin{bmatrix}x_1\\ x_2\end{bmatrix}$, $A = \begin{bmatrix}7 & 11\\ 10 & 8\\ 1 & 0\\ 0 & 1\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}77\\ 80\\ 9\\ 6\end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix}150\\ 175\end{bmatrix}$, the canonical form of the problem is

$$\max\left\{\mathbf{c}^{\mathrm{T}} \mid A\mathbf{x} \le b \land \mathbf{x} \ge 0\right\}$$

(b) Due to the absense of `linprog` in Octave, we instead use GNU GLPK:

```
octave> x = glpk (c, A, b, [], [], "UUUU", "CC", -1)
x =
   4.8889
   3.8889
```

Contraint type `UUUU` is used because all contraints are inequalities with an upper bound and `CC` indicates continous values of $\mathbf{x}$. With the sense of -1, GLPK looks for the maximization[1] of $\Pi(4.8889, 3.8889) = 1413.9$.

The two blank arguments are for the lower and upper bounds of $\mathbf{x}$, default to zero and infinite respectively. Alternatively we can use the following to obtain the same result

```
glpk (c, [7 11; 10 8], [77; 80], [], [9 6], "UU", "CC", -1)
```

(c) Within the constrains, the profit can be calculated using the following function, which takes two meshes of x and y as arguments

```
function z = profit (x, y)
  A = [7 11; 10 8];
  b = [77; 80];
  c = [150; 175];
  [m n] = size (x);
  z = -inf (m, n);
  for s = 1 : m
    for t = 1 : n
      r = [x(s, t); y(s, t)];
      if A * r <= b
        z(s, t) = dot (c, r);
      endif
    endfor
  endfor
endfunction
```

Using this, the solution space is then plotted using `ezsurf`, which color each grid by their relative values (the smallest is dark purple and the largest is bright yellow):

```
ezsurf (@(x1, x2) constraints (x1, x2), [0 9 0 6], 58)
```

[1] I believe *minimization* was a typo in the assignment, since it is trivial that Π has the minimum value of 0 at $\mathbf{x} = \mathbf{0}$.

Since the plot is just a part of a plane, we can rotate it for a better view without losing any information about its shape.